UGEB2530 Game and strategic thinking
Quiz 2

Name:______________________________ID:_______________Marks:________/40
Time allowed: 60 mins
Answer all questions.

1. (6 marks) The game of nim is played as follows. There are 3 piles of coins. Two players remove any number of coins from any one of the piles alternatively. The player who removes the last coin wins.

   (a) Find $14 \oplus 12 \oplus 7$ where $\oplus$ denotes the nim-sum.
   Solution:

   (b) Find all winning move of the first player if initially the number of coins in the three piles are $14, 12, 7$.
   Solution:

2. (6 marks) Explain whether the following bimatrix games can be transformed to a zero sum game.

   (a) $\begin{pmatrix} (1,0) & (-1,1) \\ (-3,2) & (5,-2) \end{pmatrix}$

   Solution:

   (b) $\begin{pmatrix} (3,5) & (-1,-3) \\ (1,1) & (2,3) \end{pmatrix}$

   Solution:

3. (6 marks) Circle all pure Nash equilibria of the games with the following game bimatrices and state whether they are Pareto optimal.

   (a) $\begin{pmatrix} (2,3) & (0,-3) \\ (-1,-1) & (3,5) \end{pmatrix}$

   (b) $\begin{pmatrix} (3,2) & (2,-2) & (4,-1) \\ (0,5) & (-2,2) & (4,3) \\ (1,-3) & (5,1) & (2,-2) \end{pmatrix}$

4. (6 marks) Consider the 2-person game with the following bimatrix

$$\begin{pmatrix} (3,1) & (0,3) \\ (-1,2) & (1,0) \end{pmatrix}$$

   (a) Find a prudential strategy for each of the players.

   (b) Find the Nash equilibrium of the game and the corresponding payoffs to the two players.

   Solution:

5. (6 marks) Find the threat solution, that is find the threat strategy and the payoff of each player, of the game with the game bimatrix

$$\begin{pmatrix} (2,3) & (4,-1) \\ (5,2) & (2,1) \end{pmatrix}$$

Solution:

6. (10 marks) A game theorist has been invited to give a talk at each of three distant cities A, B and C. He requests the host cities for the travel expenses. Since the three cities are relatively close, travel expenses can be greatly reduced if he accommodates them all in one trip. The problem is to decide how the travel expenses should be split among the three host cities. The one-way travel expenses among these three cities and his home town H are given in the following table.

Between	One-way travel expenses
H and A	7
H and B	8
H and C	6
A and B	2
A and C	4
B and C	4

For coalition $S \subset \{A, B, C\}$, let $\nu(S)$ be the among saved for coalition S. For example, if $S = \{A, B\}$, then the expenses for the game theorist to travel to A and B separately is $2 \times 7 + 2 \times 8 = 30$ and the expenses for him to travel to A and B in one trip is $7 + 2 + 8 = 17$. Hence $\nu(\{A, B\}) = 30 - 17 = 13$.

(a) Find $\nu(\{B, C\}$, $\nu(\{A, C\}$ and $\nu(\{A, B, C\}$.

(b) Find the Shapley values of A, B, C.
(Hint: $\phi_A = \dfrac{\nu(\{A, B\}) + \nu(\{A, C\}) - 2\nu(\{B, C\}) + 2\nu(\{A, B, C\})}{6}$)

(c) How should the three cities divide the travel expenses?

Solution:

-End of Paper-